

CONSIDERATIONS FOR IMPLEMENTATION

Timing

Motivation

Management Visibility

Supervisory Motivation

Personal Motivation

Background of Enrollment

Function of Group

Discipline

Course Content

Measures

Initial Poll

Final Poll

⑤ $f(t) = 3 \frac{d^2x}{dt^2} + 7 \frac{dx}{dt} + 2x$

helpful to use lower conv for time domain - easier for complex frequency, s domain

$F(s) = X(s) \{3s^2 + 7s + 2\}$

② impulse $F(s) = 1 =$

zero initial conditions

$X(s) = \frac{1}{3s^2 + 7s + 2} = \frac{1}{(3s+1)(s+2)} = \boxed{\frac{1/3}{(s+1/3)(s+2)}}$

transfer function $H(s)$

$\boxed{\frac{X(s)}{F(s)} = \frac{1/3}{(s+1/3)(s+2)}}$

impulse response = transfer function expression

⑥ unit step $F(s) = 1/s$

zero initial conditions

$X(s) = \frac{1/3}{(s)(s+2)(s+1/3)}$

$= \frac{1}{3} \left[\frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+1/3} \right]$ partial fraction expansion

$\frac{(s+1/3)A(s+2) + B(s)(s+1/3) + C(s)(s+2)}{(s)(s+2)(s+1/3)} = \frac{1}{(s)(s+2)(s+1/3)}$

$s = -1/3$

$s = 0$

$s = -2$

$A(1/3)(2) = 1$

$B(-2)(-5/3)$

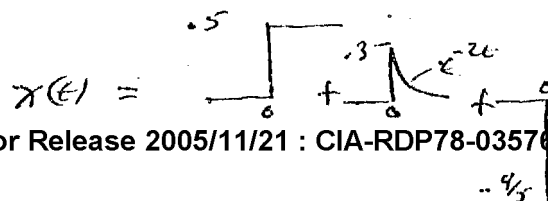
$C(-1/3)(-5/3+2) = 1, C = -9/5$
 $A = 3/2$
 $B = 3/10$

$\boxed{X(s) = \frac{1}{3} \left\{ \frac{3/2}{s} + \frac{3/10}{s+2} - \frac{9/5}{s+1/3} \right\}}$

⑥

$x(t) = \frac{1}{3} \left\{ \frac{3}{2} + \frac{3}{10} e^{-2t} - \frac{9}{5} e^{-t/3} \right\}$

check $\lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} sX(s) = 0$
 $\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sX(s) = 1/2$



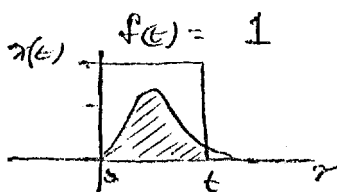
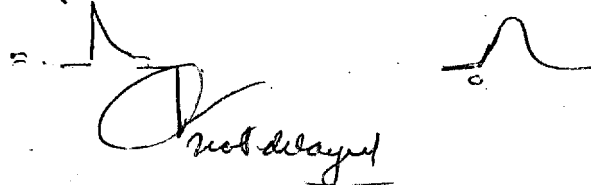
④ impulse response = $H(s) = \frac{1}{3} \left\{ \frac{A}{s + \frac{1}{3}} + \frac{B}{s + 2} \right\}$

$$\frac{B(s + \frac{1}{3}) + A(s + 2)}{(s + \frac{1}{3})(s + 2)} = \frac{1}{(s + \frac{1}{3})(s + 2)}$$

$$\begin{array}{lll} s = -\frac{1}{3} & A \frac{5}{3} = 1 & A = \frac{3}{5} \\ s = -2 & B(-\frac{5}{3}) = 1 & B = -\frac{3}{5} \end{array}$$

$$H(s) = \left\{ \frac{1/5}{s + \frac{1}{3}} + \frac{1/5}{s + 2} \right\}$$

$$h(t) = \frac{1}{5} \left\{ e^{-t/3} - e^{-2t} \right\}$$



$$c(t) = f(t) * h(t) = \int_{-\infty}^{\infty} h(\tau) f(t - \tau) d\tau$$

$$= \frac{1}{5} \int_0^t (e^{-\tau/3} - e^{-2\tau}) d\tau$$

$$= \frac{1}{5} \left[\frac{e^{-\tau/3}}{-1/3} - \frac{e^{-2\tau}}{-2} \right]_0^t$$

$$= \frac{1}{5} \left[-\frac{e^{-t/3} + 1}{1/3} - \frac{e^{-2t} - 1}{-2} \right]$$

$$= \frac{1}{5} \left[-3e^{-t/3} + 3 + \frac{1}{2}e^{-2t} - \frac{1}{2} \right]$$

$$x(t) = \boxed{\frac{1}{2} - \frac{3}{5}e^{-t/3} + \frac{1}{10}e^{-2t}}$$

$$X(s) = \frac{1/2}{(s)} + \frac{1/10}{(s+2)} - \frac{3/5}{(s + \frac{1}{3})}$$

To solve:

$$\frac{dY}{dx} = f(x, Y) \quad \text{on } \{a \leq x \leq b\} \quad , \quad Y(a) = Y_0$$

for $Y(x) \in C^1$, $f \in C$;

f Lipschitz cont: $|f(x, y) - f(x, y^*)| \leq L|y - y^*|$.

Usual approach is to discretize. ~~that is~~

$$x_0 = a, x_1, x_2, \dots, x_n = b$$

$$y_i = y(x_i), \quad \text{for } i = 0, 1, 2, \dots, n$$

Intuitively, use one of:

$$(I) \quad \left. \frac{dY}{dx} \right|_{x_i} \approx G(Y(x_0), Y(x_1), \dots)$$

$$\text{so solve } G_i(Y(x_0), \dots, Y(x_i)) = f(x_i, Y(x_i)) \quad , \quad i = 0, \dots, n$$

$$\left[\begin{array}{l} \text{e.g. } \cancel{y(x_{k+1}) = y(x_k) + y'(x_k)(x_{k+1} - x_k) + \dots} \\ \text{so } y'(x_k) \approx \frac{y(x_{k+1}) - y(x_k)}{x_{k+1} - x_k} \\ \text{Get } y(x_{k+1}) = y(x_k) + (x_{k+1} - x_k) f(x_k, y_k) \end{array} \right]$$

(II) Convert to integral equation:

$$y(x) = y(x_0) + \int_{x_0}^x f(x, y(x)) dx$$

and numerically integrate 

Get iteration scheme.

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 But both reduce to the same thing:

We convert the ODE into some approximate discrete form which is valid for certain kinds of functions (e.g. polys of order 3)

Since Runge-Kutta sol'n is so popular, I concentrate on Predictor-Corrector, 2nd order

$$\begin{cases} y_{n+1} = y_n + \frac{h}{2} (y'_{n+1} + y'_n) & \textcircled{1} \\ y_{n+1} = y_{n-2} + \frac{3h}{2} (y'_n + y'_{n-1}) & \textcircled{2} \end{cases}$$

$$y'_n \equiv f(x_n, y_n) \\ h = x_{n+1} - x_n = x_n - x_{n-1}, \text{ etc.}$$

~~$y(x) = ax^2 + bx + c$~~

$$\left[\begin{aligned} y(x_{n+1}) &= y(x_n) + y'(x_n)h + y''(x_n)\frac{h^2}{2} + \dots \\ y(x_n) &= y(x_{n+1}) + y'(x_{n+1})(-h) + y''(x_n)\frac{h^2}{2} + \dots \\ \Rightarrow (y'(x_n) + y'(x_{n+1}))h &= 2(y(x_{n+1}) - y(x_n)) \end{aligned} \right]$$

~~Error on $\frac{1}{12}$~~ $y'''(\xi)(h^3)\left(\frac{1}{12}\right) \text{ or } \eta \leq 1$
 $\neq \left(\frac{3}{4}\right)$

~~$\frac{1}{12}$~~
 $\textcircled{1}$
 $\textcircled{2}$

~~Use~~ $\textcircled{1}$ is implicit $\Rightarrow$ iterate
 $\textcircled{2}$ is forward.

Use $\textcircled{2}$ as "predictor"
 $\textcircled{3}$ as "corrector";

$$\textcircled{2} \quad y_{n+1}^{(0)} = y_{n-2} + \frac{3h}{2} (\cancel{f(x_n, y_n)} + f(x_{n-1}, y_{n-1})) \quad [\text{Predict}]$$

$$\textcircled{1} \quad y_{n+1}^{(j+1)} = y_n + \frac{h}{2} \{ f(x_{n+1}, y_{n+1}^{(j)}) + f(x_n, y_n) \} \quad [\text{Correct}]$$

Starting

Use Runge-Kutta or:

Adams-Bashforth Predictor

$$y_{n+1} = y_n + h \cancel{f(x_n, y_n)}$$

$$y_{n+1} = y_n + h/2 [3 \cancel{f(x_n, y_n)} - f(x_{n-1}, y_{n-1})]$$

$$y_{n+1} = y_n + \frac{h}{12} [23 f(x_n, y_n) - 16 f(x_{n-1}, y_{n-1}) + 5 f(x_{n-2}, y_{n-2})]$$